PHYSICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 52

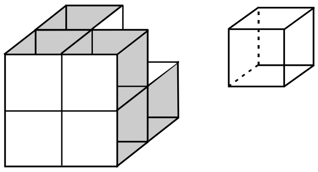
Total Marks : 24

Max. Time : 25 min.

Topics : Center of Mass, Work, Power and Energy

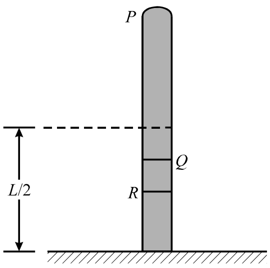
Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.4	(3 marks, 3 min.)	[12, 12]
Multiple choice objective ('–1' negative marking) Q.5 to Q.6	(4 marks, 4 min.)	[8, 8]
Subjective Questions ('–1' negative marking) Q.7	(4 marks, 5 min.)	[4, 5]

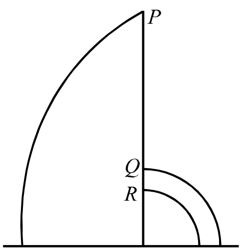
1. 8 small cubes of length ℓ are stacked together to form a single cube. One cube is removed from this system. The distance between the centre of mass of remaining 7 cubes and the original system is :



(A) $\frac{7\sqrt{3}\,\ell}{16}$ (B) $\frac{\sqrt{3}}{16}\ell$ (C) $\frac{\sqrt{3}}{14}\ell$ (D) zero

2. A uniform rod of mass M and length L falls when it is made to stand on a smooth horizontal floor. The trajectories of the points P, Q and R as shown in the figure given below is best represented by :



(A)


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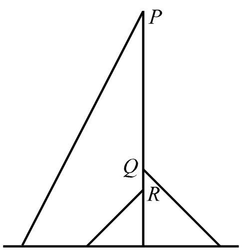
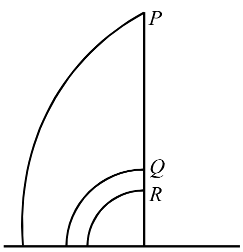
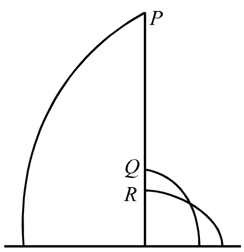


(C)


(D)


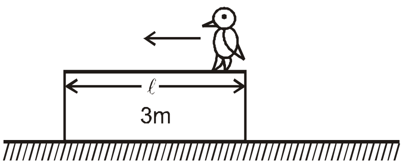
3. A man places a vertical uniform chain (of mass 'm' and length 'ℓ') on a table slowly. Initially the lower end of the chain just touches the table. The man drops the chain when half of the chain is in vertical position. Then work done by the man in this process is :

(A) $-\,mg\,\frac{\ell}{2}$ (B) $-\,\frac{mg\ell}{4}$ (C) $-\,\frac{3mg\ell}{8}$ (D) $-\,\frac{mg\ell}{8}$

4. The potential energy (in SI units) of a particle of mass 2 kg in a conservative field is U = 6x – 8y. If the initial velocity of the particle is $\vec{u} = -1.5\,\hat{i} + 2\,\hat{j}$ then the total distance travelled by the particle in first two seconds is

(A) 10 m (B) 12 m (C) 15 m (D) 18 m

5. A penguin of mass m stands at the right edge of a sled of mass 3m and legnth ℓ. The sled lies on frictionless ice. The penguin starts moving towards left, reaches the left end and jumps with a velocity u and at an angle θ relative to ground. (Neglect the height of the sled)



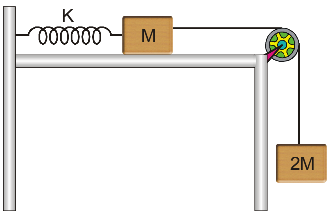
(A) Till the penguin reaches the left end, the sled is displaced by $\frac{\ell}{4}$

(B) Till the penguin reaches the left end, the sled is displaced by $\frac{\ell}{3}$

(C) After jumping, it will fall on the ground at a distance $\frac{4}{3}\frac{u^2 \sin 2\theta}{g}$ from the left end of the sled.

(D) After jumping, it will fall on the ground at a distance $\frac{3}{4}\frac{u^2 \sin 2\theta}{g}$ from the left end of the sled.

6. Two blocks, of masses M and 2M, are connected to a light spring of spring constant K that has one end fixed, as shown in figure. The horizontal surface and the pulley are frictionless. The blocks are released from rest when the spring is non deformed. The string is light.



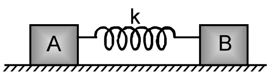
(A) Maximum extension in the spring is $\frac{4Mg}{K}$.

(B) Maximum kinetic energy of the system is $\frac{2M^2g^2}{K}$

(C) Maximum energy stored in the spring is four times that of maximum kinetic energy of the system.

(D) When kinetic energy of the system is maximum, energy stored in the spring is $\frac{4M^2g^2}{K}$

7. In the figure shown the spring is compressed by 'x_0' and released. Two blocks 'A' and 'B' of masses 'm' and '2m' respectively are attached at the ends of the spring. Blocks are kept on a smooth horizontal surface and released. Find the work done by the spring on 'A' by the time compression of the spring reduced to $\frac{x_0}{2}$.



Answers Key

DPP NO. - 52

1. (C) **2.** (D) **3.** (C)
4. (C) **5.** (A), (C) **6.** (A), (B), (C)

7. $\frac{1}{4}\, k\, x_0^2$

Hint & Solutions

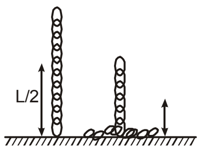
DPP NO. - 52

1. $7M\,(x\hat{i} + y\hat{j} + z\hat{k}) = M\left(\frac{\ell}{2}\hat{i} + \frac{\ell}{2}\hat{j} + \frac{\ell}{2}\hat{j}\right)$

Shifting = $\sqrt{x^2 + y^2 + z^2} = \sqrt{3}\,.\,\frac{\ell}{\sqrt{14}}$

2. Path of Q and R will intersect and will be on opposite to that of P .
Since there is no friction, the centre of mass will fall vertical downward. When the rod falls on the ground, it is shown as a dotted.

3. **(C)** The work done by man is negative of magnitude of decrease in potential energy of chain



$\Delta U = mg\,\frac{L}{2} - \frac{m}{2}g\,\frac{L}{4} = 3\,mg\,\frac{L}{8}$

$\therefore\quad W = -\,\frac{3mg\ell}{8}$

4. $= -\frac{\partial U}{\partial x}\hat{\ } - \frac{\partial U}{\partial y}\hat{j} = -[6\hat{\ }] + [8]\hat{j}$

$= -6\hat{\ } + 8\hat{j}$

$\therefore\ a = -3\hat{\ } + 4\hat{j}$ has same direction as that of

$\vec{u} = \frac{-3\hat{i} + 4\hat{j}}{2} = \left(\frac{a}{2}\right)$

$|a| = 5$

$|u| = 5/2$

Since u and a are in same direction, particle will move along a straight line

$\therefore\ S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2$

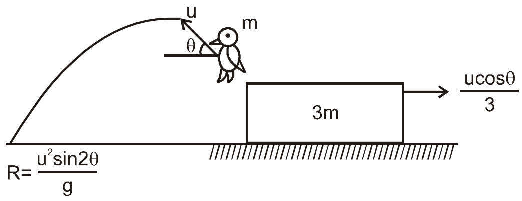
$= 5 + 10 = 15$ m. **15 m. Ans**

5. (A) $S_m = \frac{m_1 S_1 + m_2 S_2}{m_1 + m_2}$

$0 = \frac{(3m)(-x) + (m)(\ell - n)}{3m + m}$

$x = \frac{\ell}{4}$

(C)



$T = \frac{2u\sin\theta}{g}$

Displacement of sled in this time =

$\left(\frac{u\cos\theta}{3}\right)\left(\frac{2u\sin\theta}{g}\right) = \frac{1}{3}\left(\frac{u^2\sin 2\theta}{g}\right)$

Total distance $= \frac{4}{3}\left(\frac{u^2\sin 2\theta}{g}\right)$

6. Maximum extension will be at the moment when both masses stop momentarily after going down.

Applying W-E theorem from starting to that instant.

$k_f - k_i = W_{gr.} + W_{sp} + W_{ten}.$

$$0 - 0 = 2\,M.g.x + \left(-\frac{1}{2}Kx^2\right) + 0$$

$$x = \frac{4Mg}{K}$$

System will have maximum KE when net force on the system becomes zero. Therefore

2 Mg = T and T = kx

$$\Rightarrow \quad x = \frac{2Mg}{K}$$

Hence KE will be maximum when 2M mass has gone down by $\frac{2Mg}{K}$.

Applying W/E theorem

$$k_f - 0 = 2Mg. \qquad - \frac{1}{2}K.\frac{4M^2g^2}{2}$$

$$k_f = \frac{2M\ g}{2}$$

Maximum energy of spring = $\frac{1}{2}K.\left(\frac{4Mg}{K}\right)$

$$= \frac{8M\ g}{}$$

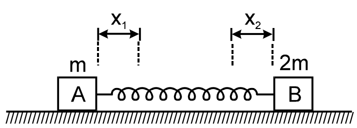
Therefore Maximum spring energy
= 4 × maximum K.E.

When K.E. is maximum $x = \frac{2Mg}{K}$.

$$\text{Spring energy} = \frac{1}{}.K.\frac{4M\ g}{2} = \frac{2M\ g}{2}$$

i.e. (D) is wrong.

7.



Let the block A shift to left by x_1 and block B shift to right by x_2. The centre of mass of the two block system is at rest

Hence $mx_1 = 2mx_2$

or $x_2 = \frac{x_1}{2}$ ....(1)

and the spring force on either block is $k(x_0 - [x_1 + x_2])$, where x_0 is the initial compression in the spring

Let the block A shift further left by dx_1

∴ work done on block by spring is

$dW = k(x_0 - x_1 - x_2)\,dx_1$ ....(2)

$= k\left(x_0 - x_1 - \frac{x_1}{2}\right)dx_1$

$dW = k\left(x_0 - \frac{3}{2}x_1\right)dx_1$

∴ Net work done

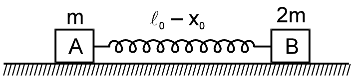
$$\int dW = \int_{x_1=0}^{x_0/3} k\left(x_0 - \frac{3}{2}x_1\right)dx_1 = \frac{k\,x_0^2}{4}$$

Ans. $\frac{1}{4}k\,x_0^2$

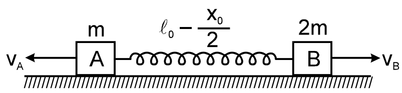
ALTERNATIVE SOLUTION

Let the speeds of blocks A and B at the instant compression is $\frac{x_0}{2}$ be v_A and v_B as shown in figure

[ℓ_0 = natural length of spring]



Initial state



Final state

No external forces act on the system in the horizontal direction

Applying conservation of momentum in horizontal direction

initial momentum = final momentum

$0 = m(-v_A) + 2m\,v_B$

or $v_A = 2v_B$ ....(1)

from conservation of energy

$$\frac{1}{2}k\,x_0^2 = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 + \frac{1}{2}m\,v_A^2 + \frac{1}{2}2m\,v_B^2$$

..........(2)

from (1) and (2) we get

$$\frac{1}{2} m v_A^2 = \frac{1}{4} k x_0^2$$

work done on block A by spring = change in kinetic energy of block A

$$= \frac{1}{2} m v_A^2 = \frac{1}{4} k x_0^2$$